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Dependence Measures in Malaysian Stock Market

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ABSTRACT

Return series tend to have leptokurtic distribution thus making linear correlation an inappropriate measure of dependence. The copula has been used to capture such dependency. However, there has been little literature on modeling and estimating the dependence between two series in Malaysian financial stock market. The purpose of this study is to investigate the dependence structure between two return series of the Kuala Lumpur Composite Index (KLCI) and the capitalization weighted index (EMAS) for the period 1998:M1 to 2005:M12. The results show that the Student's *t* copula is suitable to represent the dependence structure of KLCI-EMAS pair. This finding indicates that there is slight strong dependence at the upper and lower tails of the two series. The significance of these tail dependences implies that the return series of Kuala Lumpur Composite Index and the capitalization weighted index tend to experience concurrent shocks.

Keywords: copula, tail dependence, dependence structure.

1. INTRODUCTION

The most used tool to measure dependence between two returns series in any financial stock market is linear correlation. This association measure is an appropriate measure if the returns follows normal distribution. However, in reality, the behavior of returns hardly follows this assumption (Ruzanna and Noriszura, 2014). Return series have been described to have skewed distributions and fat tails. These facts make correlation an inappropriate measure of dependence as it can lead to the underestimation of risk of a portfolio (Penzer, Schmid and Schmidt (2012); Zhou and Gao (2012)).

The Copula can be used an alternative to linear correlation. It is known of its ability to capture the dependence structure either symmetric or asymmetric distribution (Righi and Ceretta (2011); Shamiri, Nor Aishah and Pirmoradian (2011)). It is also able to provide the degree of dependence. Furthermore, the dependence structure can be modeled separately from the user-specified marginal models (Ning (2010)). In Malaysia, there has been little literature on modeling and estimating dependence in financial stock markets.

The purpose of this study is to investigate the dependence structure between two return series in Malaysian stock market. The outcomes of this study will provide better understanding of using copula modes in estimating the dependence between the series. The following subsection introduces a brief back ground of copula and the families that are considered in our study. The remaining sections are methodology, results and conclusion of this study.

1.1 Copula

A copula is a function of the marginal distribution of each event that returns the joint distribution of the events. For a bivariate case, a copula function can be expressed as,

$$C(u,v) = C[F(x), G(y)] \tag{1}$$

where $C(u,v)$ is defined as a copula function which relates the marginal distribution functions $F(x)$ and $G(y)$ into their joint probability distribution. The concept of copula was first introduced by Sklar (1959) and then supported by Joe (1997) and Nelson (2006). In this study, we limit our investigation with five most commonly used copula models which belongs to the Archimedean and Elliptical copula families.

1.2 Archimedean Copulas

In our study, we considered the Clayton, Gumbel and Frank copula models. The relationship between the Clayton copula parameter, α, and Kendall's tau, τ, is given by:

$$\hat{\alpha} = \frac{2\tau}{1-\tau} \quad (2)$$

The relationship between the Gumbel copula parameter, α, and Kendall's tau, τ, is given by:

$$\hat{\alpha} = \frac{1}{1-\tau} \quad (3)$$

The relationship between the Frank copula parameter, α, and Kendall's tau, τ, is given by:

$$\frac{D_1(\alpha)-1}{\alpha} = \frac{1-\tau}{4} \quad (4)$$

where $D_1(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} \frac{1}{e^{\tau}-1}\, dt$ is a Debye function.

1.3 Elliptical Copulas

The Gaussian (normal) copula and Student's t shares the same parametric dependence measure, ρ. The relationship between the elliptical copulas parameter, ρ, and Kendall's tau, τ, is given by:

$$\rho = \sin\left(\frac{\pi}{2}\tau\right) \quad (5)$$

However, the difference between the Gaussian copula and Student's t copula is the ability to quantify the tail dependence. The Gaussian copula is unable to capture the strength of dependence at the tails compared to Student's t copula.

2. METHODOLOGY

The data used in our study are daily closing price indices of the Kuala Lumpur Composite Index (KLCI) and the capitalization weighted index (EMAS) for the period from early January 1998 to the end of December 2005. These 1962 price indices were transformed to daily returns based on,

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \quad \text{where } t = 1, 2, \ldots, N \quad (6)$$

The return series were statistically and visually examined. We fit our data to an appropriate marginal model and examine its residuals. Several model misspecification tests including Ljung-Box and Lagrange Multiplier (LM) test were performed in order to ensure the suitability of the marginal model.

Once serial independence have been detected, the data were transformed to pseudo observations $U_i = (U_1, U_2, ..., U_n)$ where $U_1 = (u_1, v_1)$. Both u_i and v_i values were based on rankings of standardized residuals. We quantify the dependence by estimating the parameters of each copula family through the inversion of Kendall's tau (τ) which is a rank-based measure of dependence (Nelson (2006); Rong and Trück (2010)).

Finally, the goodness-of-fit test is employed to determine which copula model best describe the dependence structure of the residuals of returns. It tests the null hypothesis that the data is fitted by a copula model. The test statistic use the Cramér-Von Mises statistic given by,

$$S_n = \sum_{i=1}^{n} \left[\hat{C}^{emp}(U_i) - C_\theta(U_i) \right]^2 \quad (7)$$

The approximate p-values for this test statistic is obtained by using the parametric bootstrap provided in the software used.

2.1 Marginal model

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is adopted because it can model financial time series with conditional heteroscedastic errors (Bollersev (1986)). GARCH models have provided the best fit in assessing the effect of volatility in the dependence structure (Ning, (2010); Angabini and Wasiuzzaman (2011); de Melo Mendez and Aíube, (2011)). The GARCH model with p-lagged GARCH terms and q lagged ARCH terms is given by,

$$\begin{aligned} X_t &= \mu_x + \phi_{1x} X_{t-1} + \varepsilon_t \\ \varepsilon_t &= \sigma_t Z_t \quad \text{with } Z_t \sim iid(0,1) \\ \sigma_t^2 &= \kappa + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 \end{aligned} \quad (8)$$

where $\kappa, \alpha_j, \beta_i \geq 0$ and $\sum_{i=1}^{p} \beta_i + \sum_{j=1}^{q} \alpha_j < 1$. X_t is the daily returns of a stock at time t and σ_t^2 is the variance of ε_t.

3. RESULTS

We examine the daily returns of two stock price indices: the KLCI and EMAS index, for the period from early January 1998 to the end of December 2005. Table 1 summarizes the descriptive statistics of the return series.

TABLE 1: Summary Statistics of Daily Log Returns

	Mean	Median	Max	Min	SD	Skewness	Kurtosis
KLCI	0.00023	-0.00003	0.2082	-0.2415	0.01701	0.70374	53.29067
EMAS	0.00017	-0.00049	0.1877	-0.2063	0.01614	0.99583	41.38855

The median of the daily returns of KLCI and EMAS are both negative values. This findings imply that both stocks offer lower returns. The means daily returns of both stocks are both close to zero indicating that the average return is almost constant over time. The standard deviation for KLCI returns is slightly more volatile over time compared to the returns of EMAS. Furthermore, the results show that both return series exhibit positive skewness and have fat tails. This findings suggest that the return series are non-normally distributed.

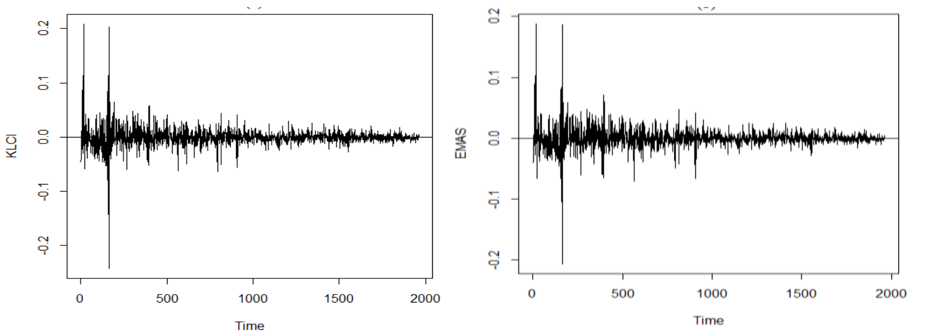

Figure 1: Time Series Plot of Daily Returns of (A) KLCI and (B) EMAS.

Figure 1 above illustrates the returns series of KLCI and EMAS stocks. Through visual inspection, we can observe that there is some volatility occurring in both stocks at the beginning of 1998. These occurrences are most likely be the after-effects of the Asian financial crisis that hit many Asian countries in 1997. It can also be seen that another shocking occurrence took place at the end of 1998. This occurrence is probably due to the political events in Malaysia. From 1999 to 2005, the return series of both stocks seems to be getting more stable. The return series were fitted into an autoregressive model and the residuals series generated were examined (Table 2). Supposedly, the residuals series should follow normal distribution and should also be serially independent.

TABLE 2: Summary Statistics of Residuals and Ljung-Box Q Statistic Estimated by AR Model

	KLCI	**EMAS**
Mean	0.00000	0.00000
Median	-0.00034	-0.00027
Standard deviation	0.01525	0.01456
Skewness	-0.11268	0.02628
Kurtosis	35.63816	30.41924
Q(10)	0.22430	0.38210
$Q^2(10)$	*1833.29600	*1570.20700

*The values are significant at 5% and 1% levels.

Based on Table 2 above, the small value of standard deviation (SD) indicates that there is some variability in the residuals. Furthermore, the distributions of these residual series are leptokurtic (non-normal). This claim is supported by the evidence of skewness and excess kurtosis as shown in Table 2. To test for serial independence and detection of the presence of conditional heteroscedastic errors, the Ljung-Box test is performed on residuals and squared residuals at lag 10. The insignificant values imply that the residuals are independent and identically distributed. However, the significant values of the Ljung-Box test on the squared residuals indicate that there exist volatility clustering around the residuals. Hence, the residuals are fitted to an ARCH(1,0) model and GARCH(1,1) model.

TABLE 3: Ljung-Box Test and Lagrange Multiplier Test on Residuals Estimated by ARCH and GARCH

		KLCI	**EMAS**
ARCH Model	Q(10)	*71.35212	*60.14108
	$Q^2(10)$	*386.45400	*276.87980
	LM test	*234.65110	*185.25610
GARCH Model	Q(10)	*97.14764	*75.97880
	$Q^2(10)$	12.58734	19.05184
	LM test	13.88974	17.81905

*The values are significant at 5% and 1% levels.

Table 3 shows the results of the Ljung-Box test on the residuals and squared residuals and also the Lagrange Multiplier (LM) test. For ARCH model, the significant values of the tests connotes that the residuals are serially correlated. Thus, ARCH is not an appropriate model to capture conditional heteroscedastic errors. For GARCH model, the insignificant values of the Ljung-Box test on the square residuals indicate that there is no variability in the residuals. Furthermore, the results of the LM test suggest that the marginal conditional distribution functions (CDFs) are now serially independent. Therefore, the GARCH model appears to be an adequate fit to model volatility.

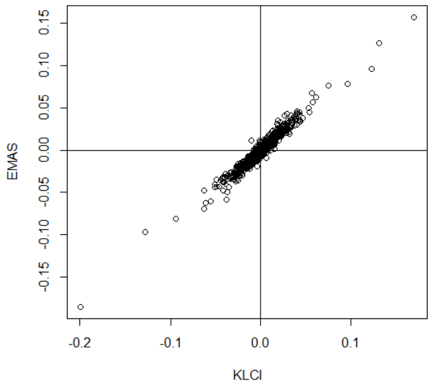

Figure 1: Bivariate Scatter Plot of the Residuals Estimated by GARCH Model.

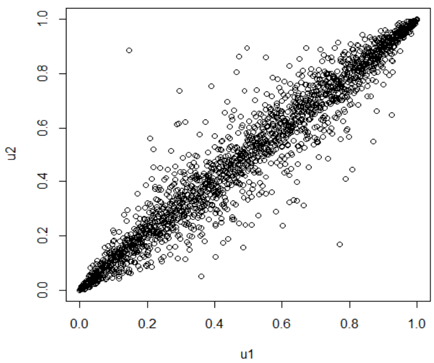

Figure 2: Bivariate Scatter Plot of U_i.

Figure 2 illustrates the scatter plot of residuals of KLCI and EMAS stocks. We can observe that there is strong dependence between the residual series. The residuals estimated by GARCH model are then transformed to paired observations $U_1 = (u_1, v_1)$. Figure 3 displays the scatterplot of the paired observations. It can be seen that there is somewhat strong dependence at the lower and upper tails of the observations. The estimated Kendall's Tau of 0.8505 implies that there is a strong dependence between KLCI and EMAS stocks.

TABLE 4: Estimated Parameters of the Copulas and the Goodness-of-fit Statistics

Copula Family	Parameter Estimates	S_n	*p*-value
Normal	0.9725	0.0117	0.0415
Student's t	0.9725	0.0082	0.2922
Clayton	11.3744	0.2662	0.0005
Gumbel	6.6872	0.0370	0.0005
Frank	24.9880	0.1009	0.0005

Through the inversion of Kendall's Tau, we estimated the parameters of Normal, Student's t, Clayton, Gumbel and Frank copulas. The results of the estimated copula parameters and the goodness-of-fit test are shown in Table 4. We found that only Student's t copula had significant S_n. This implies that the Student's t copula is an adequate fit for representing the dependence structure of KLCI and EMAS stocks.

4. CONCLUSION

The aim of this paper was to investigate the dependence structure between two return series in Malaysian stock market: KLCI and EMAS. The return series exhibited a leptokurtic distribution. We further examined the dependence structure of the two stocks and found that there was slight strong dependence at the lower and upper tails. The significance of this dependency implies that both KLCI and EMAS tend to experience concurrent shocks. Future research may include longer periods for the daily returns, use advanced marginal models and also apply mixed copula families such as Joe-Clayton and Clayton-Gumbel copulas.

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